Abstract

Magic squares are among the most popular mathematical reactions. Over the years a number of generalizations have been proposed. In the early 1960's Sedlacek asked whether " Magic" ideas could be applied to graphs.

Shortly afterwards, Kotzig and Rosa formulated the study of graph labeling or *Valuations* as they were first called. A labeling is a mapping whose domain is some set of graph elements, the set of vertices for example, or the set of all vertices and edges and whose range was a finite set of positive integers. Various restrictions can be placed on the mapping. The case that we shall find more interesting is where the domain is the set of all vertices and edges of the graph and the range consists of positive integers up to the number of vertices and edges. No repetitions are allowed.

In particular one can ask whether the set of labels associated with any edge i.e the label on the edge itself and the labels on its endpoints always add up to the same sum. Kotzig and Rosa called such a labeling as magic labelings and the graph possessing it *magic graphs.*

For a graph $G(V, E)$ an one-to-one map f from $V \cup E$ onto the set $\{1, 2, \cdots, |V|+|E|\}$ is a *vertex-magic total labeling* if there is a constant k, so that for every vertex x, $f(x) + \sum f(xy) = k$ where the sum is taken over all vertices y which are adjacent to x. The constant k is called the *magic constant* for the labeling f.

A graph $G(V, E)$ of order v and size e is called (a, d)-vertex-antimagic total if there exists a bijective function $f : V \cup E \rightarrow \{1, 2, \cdots, v + e\}$, such that the set $\{f(x) + \sum\limits_{xy \in E(G)} f(xy)\}$ where the sum is over all vertices y adjacent to x for all $x \in V(G)$

is $\{a, a+d, \cdots, a+(v-1)d\}$. In the case when the vertex labels are from the set $\{1, 2, \cdots, v\}$, (a, d)-vertex-antimagic total labeling is called a *super* (a, d)*-vertex-antimagic total labeling.*

A graph $G(V, E)$ of order v and size e is called (a, d)-edge-antimagic total if there exists a bijective function $f : V \cup E \rightarrow \{1, 2, \cdots, v+e\}$, such that the set $\{f(x)+f(y)+(xy)\}$ where $xy \in E(G)$ is $\{a, a+d, \cdots, a+(v-1)d\}$. In the case when the vertex labels are from the set $\{1, 2, \cdots, v\}$, (a, d)-edge-antimagic total labeling is called a *super* (a, d)*-edge-antimagic total labeling.*

For a simple and connected graph $G = (V(G), E(G))$, and any two vertices u and v of G $d(u, v)$ represents the distance between them and $diam(G)$ is used to indicate the diameter of G. A multi-level distance labeling is a one-to-one mapping $f : V(G) \rightarrow \mathbb{Z}^+$ satisfying the condition

$$d(u, v) + |f(u) - f(v)| \geq \ diam(G) + 1 \tag{0.0.1}$$

for every $u, v \in V(G)$. The *span* of a multi-level distance labeling f is the maximum integer in the range of f. The *multi-level distance number* or (*radio number*) of G, $rn(G)$, is the lowest span in all multi-level distance labelings of the graph G.

The aim of this work is to find the *vertex-magic total labeling*, (a, d)*-vertex-antimagic total labeling*, (a, d)*-edge-antimagic total labeling*, *super* (a, d)*-vertex-antimagic total labeling* and the *multi-level distance labeling* for different families of graphs. We also find condition on the order of some families of graphs such that they do not admit any *vertex-magic total labeling.*