Abstract

Inequalities are one of the most important instruments in many branches of mathematics such as functional analysis, theory of differential and integral equations, interpolation theory, harmonic analysis, probability theory, etc. They are also useful in mechanics, physics and other sciences. A systematic study of inequalities was started in the classical book [31] and continued in [54, 55]. In the eighties and nineties of the last century an impetuous increase of interest in inequalities took place. One result of this fact was a great number of published books on inequalities (see e.g. [4, 5, 37, 39, 38]) and on their applications (see e.g. [2, 11]). Nowadays the theory of inequalities is still being intensively developed. This fact is confirmed by a great number of recent published books (see e.g. [6, 56]) and a huge number of articles on inequalities. Thus, the theory of inequalities may be regarded as an independent area of mathematics. This PhD thesis is devoted to special kind of inequalities, namely Jensen's and some its related inequalities involving Hermite-Hadamard inequality, Hardy and its limit Polya-Knopp inequality.

In the first chapter, called Introduction, some basic notions and results from theory of convex functions and theory of inequalities are being introduced along with classical results of convex functions.

In the second chapter, The weighted Jensen's Inequality for convex-concave anti-symmetric functions is proved and some applications are given.

In the third chapter we have discussed the generalized form of Hermite-Hadamard inequality for integrable Convex functions.

In the fourth chapter Some estimates of Hardy, strengthened Hardy-Knopp and

multidimensional Hardy-Polya-Knopp type differences for $p < 0$ and $0 < p < 1$ are calculated.

In the fifth chapter we prove a new general one-dimensional inequality for convex functions and Hardy-Littlewood averages. Furthermore, we apply this result to unify and refine the so-called Boas's inequality and the strengthened inequalities of the Hardy-Knopp-type, deriving their new refinements as special cases of the obtained general relation. In particular, we get new refinements of strengthened versions of the well-known Hardy and Pólya-Knopp's inequalities, while in the last chapter some measures of divergences between vectors in a convex set of $n-$dimensional real vector space are defined in terms of certain types of entropy functions, and their log-convexity properties with some applications in Information theory are discussed.