Abstract

In the first chapter we give some basic definitions from commutative algebra. We give some results obtained in recent years for the Stanley depth of multigraded S-modules, where $S = K[x_1, \ldots, x_n]$ is a polynomial ring in n indeteminantes with coefficients in a field K. We also give some results regarding the progress towards the Stanley's conjecture.

In the second chapter, we show that if $I \subset J$ be monomial ideals of a polynomial algebra S over a field. Then the Stanley depth of J/I is smaller or equal to the Stanley depth of $\sqrt{J}/\sqrt{I}$. We give also an upper bound for the Stanley depth of the intersection of two primary monomial ideals Q, Q', which is reached if Q, Q' are irreducible, $\mathrm{ht}(Q + Q')$ is odd and $\sqrt{Q}$, $\sqrt{Q'}$ have no common variables. These results are proved in my paper [23].

In the third chapter, we give different bounds for the Stanley depth of a monomial ideal I of a polynomial algebra S over a field K. For example we show that the Stanley depth of I is less than or equal to the Stanley depth of any prime ideal associated to S/I. Also we show that the Stanley's conjecture holds for I and S/I when the associated prime ideals of S/I are generated by disjoint sets of variables. These results are proved in my paper [24].

In the forth chapter, we give an upper bound for the Stanley depth of the edge ideal I of a k-partite complete graph and show that Stanley's conjecture holds for I. Also we give an upper bound for the Stanley depth of the edge ideal of an s-uniform complete bipartite hypergraph. In this chapter we also give an upper bound for the Stanley depth of the edge ideal of a complete k-partite hypergraph and as an application we give an upper bound for the Stanley depth of a monomial ideal in a polynomial ring S. We give a lower and an upper bound for the cyclic module S/I.

associated to the complete k-partite hypergraph. These results are proved in our papers [26] and [27].

In the fifth chapter, the associated primes of an arbitrary lexsegment ideal $I \subset S$ are determined. As application it is shown that S/I is a pretty clean module, therefore, S/I is sequentially Cohen-Macaulay and satisfies the Stanley's conjecture. These results are proved in my paper [25].