Abstract

For a century, one of the most famous problems in mathematics was to prove the *Four-colour theorem*. This has spawned the development of many useful tools for solving graph colouring problems. In a paper [7] Birkhoff proposed a way of tackling the four-colour problem by introducing a function $P(M, \lambda)$, defined for all positive integers λ, to be the number of proper $\lambda-$colourings of a map M. It turns out that $P(M, \lambda)$ is a polynomial in λ, called the *chromatic polynomial* of M. If one could prove that $P(M, 4) > 0$ for all maps M, then this would give a positive answer to the four-colour problem. The polynomial $P(M, \lambda)$ is defined for all real and complex values of λ.

The notion of chromatic polynomial was later generalized to that of an arbitrary graph by Whitney [52], who established many fundamental results for it. In 1968, Read (see [36]) aroused new interest in the study of chromatic polynomials with his well referenced introductory article on the subject. He asked if it is possible to find a set of necessary and sufficient algebraic conditions for a polynomial to be the chromatic polynomial of some graph. For example, it is true that the chromatic polynomial of a graph determines the numbers of vertices and edges and that its coefficients are integers which alternate in sign.

Read asked: What is the necessary and sufficient condition for two graphs to be chromatically equivalent; that is, to have same chromatic polynomial? In particular Chao and Whitehead Jr. [10] defined a graph to be chromatically unique if no other graphs share its chromatic polynomial. They found several families of such graphs.

Since then many invariants under chromatic equivalence have been found and various families of and results on such graphs have been obtained successively. The question of chromatic equivalence and uniqueness is termed the chromaticity of graphs. Although Birkhoff's hope of using the chromatic polynomial to prove the four-colour theorem was not borne out, it has attracted a steady stream of attention through the years.

In chapter 5 we will prove that the Jahangir graph is chromatically unique for $p = 3$.

Dohmen [14] and Tomescu [44] initiated and discussed the study of chromatic polynomials and chromaticity of *linear uniform hypergraphs*. Tomescu [44] obtained the coefficients of the chromatic polynomial up to the girth (length of the shortest cycle) of the hypergraphs. He also proved some results to prove that two linear uniform hypercycles having a common hyperedge is chromatically unique in the class of h-hypergraphs. Recently in a paper [45] he proved that the sunflower hypergraphs are h-chromatically unique.

In chapter 6, we will generalize the result proved in [44] related to the chromaticity of two linear uniform h-hypercycles having a path in common. Also, we will prove an important result which tells us that the number of cycles of a linear hypergraph is bounded below by its cyclomatic number. At the end, we find the chromatic polynomial of a connected linear uniform unicyclic hypergraph.