Abstract

A Halin graph is a graph $H = T \cup C$, where T is a tree with no vertex of degree two, and C is a cycle connecting the end-vertices of T in the cyclic order determined by a plane embedding of T. Halin graphs were introduced by R. Halin [16] as a class of minimally 3-connected planar graphs. They also possess interesting Hamiltonian properties. They are 1-Hamiltonian, i.e., they are Hamiltonian and remain so after the removal of any single vertex, as Bondy showed (see [23]). Moreover, Barefoot proved that they are Hamiltonian connected, i.e., they admit a Hamiltonian path between every pair of vertices [1]. Bondy and Lovász [6] and, independently, Skowronska [33] proved that Halin graphs on n vertices are almost pancyclic, more precisely they contain cycles of all lengths l $(3 \leq l \leq n)$ except possibly for a single even length. Also, they showed that Halin graphs on n vertices whose vertices of degree 3 are all on the outer cycle C are pancyclic, i.e., they must contain cycles of all lengths from 3 to n.

In this thesis, we define classes of generalized Halin graphs, called k-Halin graphs, and investigate their Hamiltonian properties.

In chapter 4, we define k-Halin graph in the following way.

A 2-connected planar graph G without vertices of degree 2, possessing a cycle C such that
(i) all vertices of C have degree 3 in G, and
(ii) $G - C$ is connected and has at most k cycles
is called a *k-Halin graph*.

A 0-Halin graph, thus, is a usual Halin graph. Moreover, the class of k-Halin graphs is contained in the class of $(k+1)$-Halin graphs ($k \geq 0$).

We shall see that, the Hamiltonicity of k-Halin graphs steadily decreases as k increases. Indeed, a 1-Halin graph is still Hamiltonian, but not Hamiltonian connected, a 2-Halin graph is not necessarily Hamiltonian but still traceable, while a 3-Halin graph is not even necessarily traceable. The property of being 1-Hamiltonian, Hamiltonian connected or almost pancyclic is not preserved, even by 1-Halin graphs. However, Bondy and Lovász' result about the pancyclicity of Halin graphs with no inner vertex of degree 3 remains true even for 3-Halin graphs.

The property of being Hamiltonian persists, however, for large values of k in cubic 3-connected k-Halin graphs. In chapter 5, it will be shown that every cubic 3-connected 14-Halin graph is Hamiltonian. A variant of the famous example of Tutte [37] from 1946 which first demonstrated that cubic 3-connected planar graphs may not be Hamiltonian, is a 21-Halin graphs. The cubic 3-connected planar non-Hamiltonian graph of Lederberg [21], Bosák [7] and Barnette, which has smallest order, is 53-Halin. The sharpness of our result is proved by showing that there exist non-Hamiltonian cubic 3-connected 15-Halin graphs.