Abstract

The continuous weak subsolutions of general type second order linear partial differential equations are studied in the present thesis.

Based on monotonic approximation techniques developed by Walter Littman (1963) we prove that under some regularity conditions on the coefficients of the uniformly elliptic differential operator any bounded continuous weak subsolution in a smooth domain D possesses all first order weak (Sobolev) partial derivatives and belongs to the weighted Sobolev space $H^1(D;h)$, where $h(x)$ is the appropriate weight function. Moreover, we establish a new type weighted reverse Poincare inequality for the difference of two bounded and continuous weak subsolutions.

Further the latter inequality is applied to the approximation problem of the gradient of the analytically unknown value function of the optimal stochastic control problem, the value function being the unique solution of the Hamilton-Jacobi-Bellman equation.