Abstract

We study two types of problems: In first two Chapters we study evolution inclusions and use discrete approximations to obtain some new qualitative results, although the technique is not essentially new. In last two Chapters, we study the numerical approximation of impulsive differential equations and impulsive functional differential equations.

In first Chapter, we study differential inclusions on evolution triple. We replace the Lipschitz condition with Kamke one which is much weaker. The general approach of Mordukhovich appears to be very flexible and up to some technical difficulties it can be used for larger class of problems. We prove a new variant of the well known lemma of Filippov–Pliss. Afterwards we extend the well known Bogolyubov's theorem and the relaxation stability of the optimal control system. Examples of control systems governed by partial differential equations are provided, where our results are applicable.

In second Chapter, we study autonomous evolution inclusions with one-sided Lipschitz right-hand side with negative constant. We prove the existence of a unique strong forward attractor and a unique strong backward attractor when the one-sided Lipschitz constant is positive. As a corollary some surjectivity and fixed point results are proved. Example of a parabolic system, satisfying our assumptions is provided.

In third Chapter, we study higher order numerical approximations of solutions

of impulsive differential equations with non-fixed times of impulses. Using a Runge-Kutta method under natural assumptions on the impulsive surfaces and the impulses we calculate good approximations of the jump times, which enables us to extend the classical results for higher order of convergence of explicit and implicit Runge-Kutta methods to more complicated systems. We provide numerical examples to show some applications of our theory. To our knowledge there are no related results in the literature of impulsive differential equations with non-fixed time of impulses.

In last Chapter, we study impulsive functional differential equations with non fixed times of impulses and their discrete approximations with the Euler's method. Under the assumption that right-hand side is Lipschitz with respect to a new norm introduced here we show the $O(h)$ approximations via Euler's method (with respect to defined metrics) of the unique solution of given delay impulsive differential system. Some examples of impulsive delay differential equations are given at the end. Notice that in general the higher-order methods are not applicable here.