Abstract

Mathematical inequalities play an important role in almost all branches of mathematics as well as in other areas of science. The basic work "Inequalities" by Hardy, Littlewood and Polya appeared 1934 and the books "Inequalities" by Beckenbach and Bellman published in 1961 and "Analytic inequalities" by Mitronovic published in 1970 made considerable contribution to this field and supplied motivation, ideas, techniques and applications. This theory in recent years has attached the attention of large number of researchers, stimulated new research directions and influenced various aspect of mathematical analysis and applications. Since 1934 an enormous amount of effort has been devoted to the discovery of new types of inequalities and the application of inequalities in many part of analysis. The usefulness of Mathematical inequalities is felt from the very beginning and is now widely acknowledged as one of the major deriving forces behind the development of modern real analysis. This Ph.D thesis deals with the inequalities for Bregman and Burbea-Rao divergences and some of its related inequalities, namely Jensen's inequality, majorization inequality, Slater's inequality and inequalities obtained by Matić and Pečarić.

The first chapter contains a survey of basic concepts, indications and results from theory of convex functions and theory of inequalities used in subsequent chapters to which we refer as the known facts.

In the second chapter we give an improvement of Jensen's inequality for convex monotone function and various applications for related inequalities and divergences.

In the third chapter we give Sapogov's extension of Čebyšev's inequality and use

this extension to prove majorization inequality. We also give mean value theorems for majorization inequality. As application, we present a class of Cauchy's means and prove logarithmic convexity for differences of power means.

In the fourth chapter we generalize some results of Matić and Pečarić. We use a log-convexity criterion and establish improvements and reverses of Slater's and related inequalities.

In the fifth chapter we give Bregman and Burbea-Rao divergences for double integrals and matrices. We derive mean-value theorems for the divergences induced by C^2-functions. As application, we present certain Cauchy type means. We prove positive semi-definiteness of the matrices generated by these divergences which implies exponential convexity and log-convexity of the divergences. Also show the monotonicity of the corresponding means of Cauchy type. At the end we consider integral power means.

In the sixth chapter we give several results for functions of two variables and majorized matrices by using continuous convex functions and Green function. We prove mean value theorems and give generalized Cauchy means. We give applications of those generalized means and show that they are monotonic. We prove positive semi-definiteness of matrices generated by differences deduced from the majorization inequalities for double integrals and majorized matrices which implies exponential convexity and log-convexity of these differences.