

This dissertation is divided into five chapters. In Chapter 1, some preliminary definitions and results from commutative and homological algebra are given. Some characterization theorems about the Noetherianess of algebras (the main topic of the writing) are listed. A subsection about the graded Rees algebras associated to the filtrations of ideals is included. Due to special interest in these algebras we give a version of Artin Rees lemma and the characterization theorem about the Noetherianess of these Rees rings. Next section is about local cohomology, we introduce some homological tools which are used to prove and support our results. In last section, we define the symmetric and factorial closure algebras associated to the modules. Finiteness or equivalently Noetherianess of the factorial closures is the central topic of this dissertation.

In Chapter 2, we define the monomial space curves and their defining prime ideals to these curves in the polynomial ring  $R = K[x, y, z]$ , where  $K$  is an arbitrary field. We summarize the work done in the 20th century to answer the question of R. Cowsik: Is the symbolic blow-up algebras  $\bigoplus_{n \geq 0} \mathfrak{p}^{(n)} T^n$ , of a prime ideal  $\mathfrak{p}$  in a complete local ring, Noetherian. This is a version of the Hilbert's fourteenth problem. We summarize the work done by many mathematicians to characterize those defining prime ideals of the monomial space curves whose symbolic blow up algebras are Noetherian. We want to extend the previous work done in the [Sc] and [KSZ] on the  $R_S(\mathfrak{p})$  for monomial ideals  $\mathfrak{p}$ , to the  $B(E)$  of the monomial modules. Some results about the finiteness of the  $R_S(\mathfrak{p})$  using the ideal transforms are proved.

Next we include a chapter about the main object of this dissertation. We define the monomial modules over the ring  $R = K[x, y, z]$ . We describe the structure of two graded  $R$ -algebras associated to these modules. Symmetric algebras and algebras whose  $n$ -th graded component is defined by the bi-dual of the  $n$ -th symmetric power of the module. We list the previously done work in this field.

In chapter 4, we describe the structure of the  $R$  algebra  $B(E)$ . We prove for a subclass of the monomial modules the  $R$ -algebra  $B(E)$  is Noetherian. It is proved that for this subclass of the monomial modules  $B(E)$  is generated as an algebra over  $R$  up to the degree 2 components. To this end the structure of the second graded component  $B(E)_2$  is required. We prove that the  $B(E)_2 / \text{Sym}_R(E)_2$  is a cyclic module, the generator of this module is explicitly defined. In this case,  $B(E)$  is a quotient of a polynomial ring over  $R$ . We describe the presentation ideal for  $B(E)$ . In this case,  $B(E)$  is a Gorenstein ring of dimension 5.

In Chapter 5, the generators of the third degree component of  $B(E)$  are described. This chapter is devoted to the question: when the algebra  $B(E)$  is generated up-to the degree 3 components. We describe that the factoriality of the algebra depends upon the characteristic of the ground field  $K$ . We prove that the subclass of monomial modules provide the examples of modules that their factorial closure algebra is a non-Cohen-Macaulay factorial ring of dimension 5 and depth 4, if the

characteristic of the field  $K$  is 2. It is one of the non-Cohen-Macaulay factorial rings of small dimensions. At the end of the last chapter, we list some topics of further research and list many interesting questions about the algebra  $B(E)$ .