Abstract

Inequalities play an important role in almost all branches of mathematics as well as in other areas of science. Books by Hardy, Littlewood and Pólya appeared in 1934 [50], Beckenbach and Bellman published in 1961 [27] and by Mitrinović published in 1970 [59] made significant contributions to the field of inequalities and provides motivations, ideas, techniques and applications. Since 1934 a huge amount of effort has been devoted to the discovery of new types of inequalities and to the applications of inequalities in many parts of analysis. The usefulness of mathematical inequalities is felt from the very beginning and is now widely acknowledged as one of the major driving forces behind the development of modern real analysis. The theory of inequalities is in a process of continuous development state and inequalities have become very effective and powerful tools for studying a wide range of problems in various branches of mathematics. This theory in recent years has attracted the attention of a large number of researchers, stimulated new research directions and influenced various aspects of mathematical analysis and applications. Among the many types of inequalities, those associated with the names of Jensen, Hadamard, Hilbert, Hardy, Opial, Poincare, Sobolev, Levin and Lyapunov have deep roots and made a great impact on various branches of mathematics. The last few decades have witnessed important advances related to these inequalities that remain active areas of research and have grown into substantial fields of research with many important applications.

The present monograph provides a systematic study of some of the most famous

and fundamental inequalities. There is no doubt that convex functions play an important role in the theory of inequalities. Many important inequalities are the consequences of the applications of convex functions. For example Jensen's inequality Hadamard's inequality for convex functions provided a good start in the development of the theory of inequalities. Some other important classes of functions are based on the definition of convex functions. Sometime it turns out to be more convenient to replace convex function with starshaped function. A convex function passing through the origin is starshaped. It is very interesting to note that now Jensen's, Hadamard's and many other inequalities have their generalizations, refinements, improvements by using as tool a class of functions which are more than convex functions. This is class of superquadratic functions introduced by S. Abramovich, G. Jameson and G. Sinnamon.

In the first Chapter we give basic notions and results briefly to provide the motivation and make the monograph friendly readable.

In Chapter 2 we give refinements of the inequalities of Aczél, Popoviciu and Bellman and some results related to power sums. We also consider divergence measures, J-divergence, K-divergence and results related to these measures are proved.

In Chapter 3 we give results related to Hadamard's and Jensen's inequalities for superquadratic functions. Then we give generalizations and improvements of these results by using converse Jensen's inequality for superquadratic functions. Also we consider refinement of Hardy's inequality and prove analogue results.

In the last Chapter we consider an Opial type integral inequality for a particular class of convex functions. We prepare results to give applications to fractional derivatives.