Abstract

"Behind every theorem lies an inequality". Mathematical inequalities play an important role in almost all branches of mathematics as well as in other areas of science. The basic work "Inequalities" by Hardy, Littlewood and Polya appeared 1934 [37]and the books "Inequalities" by Beckenbach and Bellman published in 1961 [9] and "Analytic inequalities" by Mitronovic published in 1970 made considerable contribution to this field and supplied motivation, ideas, techniques and applications. This theory in recent years has attached the attention of large number of researchers, stimulated new research directions and influenced various aspect of mathematical analysis and applications. Since 1934 an enormous amount of effort has been devoted to the discovery of new types of inequalities and the application of inequalities in many part of analysis. The usefulness of Mathematical inequalities is felt from the very beginning and is now widely acknowledged as one of the major deriving forces behind the development of modern real analysis. This dissertation deals with the inequalities for Jensen inqualites involving average of convex functions, Hermite-Hadamard inequalities. Chapter 1 offers an overview of the basic results contains a survey of basic concepts, indications and results from theory of convex functions and theory of inequalities used in subsequent chapters to which we refer as the known facts.

Chapter 2 we give proofs of convexity and Schur convexity of the generalized integral and weighted integral quasi-arithmetic mean. An overview of assorted proofs of schur convexity of integral arithmetic mean is discussed. In a detailed proof, discrete Jensen inequality for integral arithmetic mean is derived. Also integral version of Jensen inequality for integral arithmetic mean is proved. Motivated by discrete and

integral Jensen inequalities functionals are defined. Two different method is given for constructing new examples of exponentially convex functions from non trivial generating families of functions. Mean value theorem are proved. Different classes of monotonically increasing Cauchy means are created.
Chapter 3 gives us convexity and Schur convexity of functions connected to Hermite-Hadamrd inequality as well as Schur convexity of differences of Hermite-Hadamrd inequality and Hammar-Bullen inequality by different proofs. Applying assorted generalizations of Hermite-Hadamard inequality and Hammer-Bullen inequality on some special families of functions from varied classes, n-exponentially convex functions are generated by quite new method. Lyponuve, Dresher and Gramm's type inequalities are developed. Pretty different Stolarsky type means are derives preserving inherited monotonically increasing property.
Chapter 4 deals with inequalities of higher order convexity and divided difference. Two of them use majorization results and others are related to Jensen inequalities and Hermite-Hadamrd inequality. Integral Jensen inequality for divided difference is proved. Applications of averages of 3-convex functions as first order divided difference of convex functions are acquired. Method of producing n-exponentially convex functions is applied using divided differences. Produced functions are used in studying Stolarsky type means In the fifth chapter results about averages values of convex functions with variable limits and average values of composition functions is given. Study functionals for inequalities proved by D.E. Wulbert (call them Wulbert's inequalities for convenience) for convex and three convex functions. Extensions, improvements are accomplished. Variety of Stolarsky type means of a concave (convex) functions are obtained.