Abstract

Inequalities lie at the heart of a great deal of mathematics. G. H. Hardy reported Harald Bohr as saying 'all analysts spend half their time hunting through the literature for inequalities which they want to use but cannot prove'. Inequalities involving means open many doors for analysts e.g generalization of mixed means fallouts the refinements to the important inequalities of Holder and Minkowski. The well known Jensen's inequality asserts a remarkable relation among the mean and the mean of function values and any improvement or refinements of Jensen's inequality is a source to enrichment of monotone property of mixed means.

our aim is to utilize all known refinements of Jensen's inequality to give the refinements of inequality among the power means by newly defined mixed symmetric means. In this context, our results not only ensures the generalization of classical but also speak about the most recent notions (e.g n-exponential convexity) of this era.

In first chapter we start with few basic notions about means and convex functions. Then the classical Jensen's inequality and the historical results about refinements of Jensen's inequality are given from the literature together with their applications to the mixed symmetric means.

In second chapter we consider recent refinements of Jensen's inequality to refine inequality between power means by mixed symmetric means with positive weights under more comprehensive settings of index set. A new refinement of the classical Jensen's inequality is also established. The Popovicui type inequality is generalized using green function. Using these refinements we define various versions of linear functionals that are positive on convex functions. This step ultimately leads us to

the important and recently revitalized area of exponential convexity. Mean value theorems are proved for these functionals. Some non-trivial examples of exponential convexity and some classes of Cauchy means are given. These examples are further used to show monotonicity in defining parameters of constructed Cauchy means.

In third chapter we develop the refinements of discrete Jensen's inequality for convex functions of several variables which causes the generalizations of Beck's results. The consequences of Beck's results are given in more general settings. We also generalize the inequalities of Hölder and Minkowski by using the Quasiarithmetic mean function.

In forth chapter we investigate the class of self-adjoint operators defined on a Hilbert space, whose spectra are contained in an interval. We extend several refinements of the discrete Jensen's inequality for convex functions to operator convex functions. The mixed symmetric operator means are defined for a subclass of positive self-adjoint operators to give the refinements of inequality between power means of strictly positive operators.

In last chapter, some new refinements are given for Jensen's type inequalities involving the determinants of positive definite matrices. Bellman-Bergstrom-Fan functionals are considered. These functionals are not only concave, but superlinear which is a stronger condition. The results take advantage of this property.