Abstract

It is a fact that, the theory of inequalities, priding on a history of more than two centuries, plays a significant role in almost all fields of mathematics and in major areas of science. In the present dissertation, we will study the general inequalities, namely integral inequalities and discrete inequalities for generalized convex functions. Therefore, we will introduce some generalized convex functions which include functions with nondecreasing increments, $\Delta-$ and $\nabla-$convex functions, and $n-$convex functions of higher orders. By using these functions, we will provide a generalization of the Brunk's theorem, of the Levinson-type inequalities, of the Burkill-Mirsky-Pečarić's results and of the integral means. We will also discuss the Popoviciu-type characterization of positivity of sums and integrals for higher order convex functions of n variables and we will give some related results. Our dissertation also provides generalizations of some of the celebrated and fundamental identities and inequalities including Montgomery's identities, Ostrowski-, Grüss-, Čebyšev- and Fan-type inequalities. Moreover, we will also apply an elegant method of producing $n-$exponentially and logarithmically convex functions for positive linear functionals constructed with the help of majorization-type results, Favard-, Berwald- and Jensen-type inequalities. The generalization and the following refinements of Jensen-Mercer's inequalities are also provided with some applications. The Lagrange- and Cauchy-type mean value theorems are also proved and shown to be useful in studying Stolarsky-type means defined for the positive linear functionals.