Abstract

In the present thesis we investigate the almost Hermitian geometry of the twistor spaces of oriented Riemannian 4-manifolds.

Holomorphic and orthogonal bisectional curvatures have been intensively explored on Kähler manifolds and a lot of important results have been obtained in this case. But in the non-Kähler case these curvatures are not very well studied and it seems that the main reason for that is the lack of interesting examples. The first part of the thesis is devoted to the study of the curvature properties of Atiyah-Hitchin-Singer and Eells-Salamon almost Hermitian structures. This is used to provide some interesting examples of almost Hermitian 6-manifolds of constant or strictly positive holomorphic, Hermitian and orthogonal bisectional curvatures.

In the second part of the thesis we determine the Gray-Hervella classes of the so-called compatible almost Hermitian structures on the twistor spaces, recently introduced by G. Deschamps . The interest in determining these classes is motivated by the fact that the Gray-Hervella classification is a very useful tool in studying almost complex manifolds. Our results in this direction generalize the well known integrability theorems by Atiyah-Hitchin-Singer, Eells-Salamon and Deschamps and show that there is a close relation between the properties of the spectrum of the anti-self-dual Weyl tensor of an almost Kähler 4-manifold and the almost Hermitian geometry of its twistor space.