Abstract

The thesis is devoted to the weighted criteria for integral operators with positive kernels in variable exponent Lebesgue and amalgam spaces. Similar results for multiple kernel operators defined with respect to a Borel measure in the classical Lebesgue spaces are also obtained. More precisely, we established necessary and sufficient conditions on a weight function v governing the boundedness/compactness of the weighted positive kernel operator $K_v f(x) = v(x)\int_0^x k(x,y)f(y)dy$ from $L^{p(\cdot)}(\mathbb{R}_+)$ to $L^{q(\cdot)}(\mathbb{R}_+)$ under the local log-Hölder continuity condition and the decay condition at infinity on the exponents p and q. In the case when K_v is bounded but not compact, two-sided estimates of the measure of non-compactness (essential norm) for K_v are obtained in terms of the weight v and kernel k. Criteria guaranteeing the boundedness /compactness of weighted kernel operators defined on $\mathbb{R}_+$ (resp. on $\mathbb{R}$) in variable exponent amalgam spaces are found. The kernel operators under consideration involve, for example, the Riemann-Liouville transform $R_\alpha f(x) = \int\limits_0^x \frac{f(t)}{(x-t)^{1-\alpha}}dt,\ 0 < \alpha < 1$. Necessary and sufficient conditions ensuring weighted estimates for maximal and potential operators in variable exponent amalgam spaces are also established under the local log-Hölder continuity condition on exponent of spaces. Further, we establish criteria on measures governing the boundedness of integral operators with product positive kernels defined with respect to a Borel measure in the classical Lebesgue spaces. Finally, we point out that Fefferman-Stein type inequality for the multiple Riemann-Liouville transform defined with respect to a product Borel measure is derived.