

## Abstract

This dissertation deals with the Image Deblurring (ID) problem using numerical approach. ID describes the process of eliminating blur or the distortion from the blurry image. Mathematically, the connection between the captured blurry image  $z$  and a clear image  $u$  is as follow;

$$z = \vec{K}u + \eta,$$

where  $\eta$  is a noise and  $\vec{K}$  is the blur operator.

In this dissertation we purpose two mathematical models for ID problem and a splitting preconditioning. The first model in this thesis is a total fractional order variation based constrained ID model. When deblurring an image, ensuring that the intensities (pixel values) of the recovered image are strictly non-negative is essential. Nevertheless, existing numerical methods frequently fall short of yielding consistent positive outcomes, resulting in negative pixel values that significantly add the dark areas in the recovered images. To tackle this challenge, we propose a total fractional-order variational principles based mathematical model for non-blind ID. In addition to providing strictly positive intensity values, our suggested model places restrictions on the pixel values that fall within a given interval. Eliminating negative pixel values or confining them within the particular interval can greatly improve the clarity of recovered images.

The second problem in this dissertation is conformable fractional order variation based ID model. Conformable fractional derivatives provide a versatile framework for representing and utilizing the non-local and non-linear characteristics present in images. Moreover, we introduce a novel circulant preconditioner matrix that guarantees rapid convergence.

Finally we purpose splitting preconditioners for the system obtained by the discretization of conformable fractional order variation based model. This study presents novel block preconditioners based on the  $2 \times 2$  block form skew-Hermitian and Hermitian splitting for the system obtained from CFOV-based ID problem. The suggested preconditioner is based on a new partition of the  $2 \times 2$  block form, which yields

an iterative approach that is unconditionally convergent. We examine the spectral analysis of the proposed preconditioned matrices. Particularly, we illustrate that the eigenvalue spectrum of the preconditioned matrices is advantageous, thereby aiding the efficient convergence of preconditioned KSM such as GMRES.

In this research, we focus on making variational models that can clear up blurry images well without losing important details. We demonstrate the efficiency and efficacy of these models with numerical experiments.