Abstract

The theory developed about convex functions, arising from intuitive geometrical observations, may be readily applied to topics in real analysis and economics.Convexity is a simple and natural notion which can be traced back to Archimedes (circa 250 B.C.), in connection with his famous estimate of the value of π (using inscribed and circumscribed regular polygons). He noticed the important fact that the perimeter of a convex figure is smaller than the perimeter of any other convex figure surrounding it. In modern era, there occurs a rapid development in the theory of convex functions. There are sereval reasons behind it: firstly, many areas in modern analysis directly or indirectly involve the applications of convex functions; secondly, convex functions are closely related to the theory of inequalities and many important inequalities are consequences of the applications of convex functions (see [64, 47]).

Inequalities play a important role in almost all fields of mathematics. Several applications of inequalities are found in various area of sciences such as, physical, natural, engineering sciences. In numerical analysis, inequalities play a main role in error estimates of different important integrals whom analytic solutions could not be found. In recent years, a number of authors have considered extensions/generalizations of convex functions in various aspects and also tried to build several relations like Hemite-Hadamard's inequalities.

We introduce different types of convexities and drive more general Hermite-Hadamard's inequalities. Further, we derive Ostrowski, Hermite-Hadamard and Simpson, Fejer type inequalities. Also, we discuss applications of these classes such that we can estimate the integrals like $\int_a^b \frac{e^x}{x^n}dx; \int_a^b \frac{\sin x}{x^n}dx$ for $n \geq 1$ and $a, b \in (0, \infty)$ etc without

using numerical analysis.
In first chapter, we give information about convex functions, Log-convex functions, Quasi-convex functions, (s, m)-convex functions in second sense and Preinvex functions.
In second chapter, we consider the class of harmonically convex functions and investigate some relations between harmonically convex and classical convex functions. We define class of harmonically (s, m)-convex functions which unify the different harmonic convexities and establish Ostrowski, Hemite-Hadamard and Simpson, and Fejer type inequalities for this class of functions.

In third chapter, we define the classes of the harmonically p-convex functions which is a generalization of convex functions and harmonically convex functions, and harmonically p-quasiconvex functions, and harmonically logarithmic p-convex functions. Further, we investigate relationship between harmonically p-convex, p-convex and classical convex functions. Also, we give a characterization about the relation between harmonically p-convex and harmonically convex functions. Finally, we establish Hermite-Hadamard type inequalities for harmonically p-convex functions, and inequalities for the product of harmonically p-convex functions, and inequalities for harmonically logarithmic p-convex functions.

In fourth chapter, we define the class of p-preinvex functions which is generalization of preinvex and harmonically preinvex functions. We also define the notion of p-prequasiinvex and logarithmic p-preinvex functions. Moreovere, we establish Hermite-Hadamard type inequalities when the power of the absolute value of the derivative of the integrand is p-preinvex and we give results for product of two p-preinvex, and logarithmic p-preinvex functions, and Ostrowski's type for the class of p-preinvex functions.

In fifth chapter, we define harmonically (p, h, m)-preinvex functions which is generalization of harmonically preinvex functions such that preinvex and harmonically p-convex functions are its special cases. Next, we introduce the concept of harmonically logarithmic p-preinvex and harmonically p-quasipreinvex functions. Finally, we establish important and interesting results related to these classes of functions.