Abstract

In this work we study ideals generated by 2–minors of Hankel matrices which arise from combinatorics.

Let G be a closed graph, that is, a chordal graph on the vertex set $[n]$ with the maximal cliques $F_1 = [a_1, b_1], \ldots, F_r = [a_r, b_r]$ where $1 = a_1 < a_2 < \cdots < a_r < b_r = n$, and $\begin{pmatrix} x_1 & \ldots & x_{n-1} & x_n \\ x_2 & \ldots & x_n & x_{n+1} \end{pmatrix}$ the generic $2 \times n$–Hankel matrix.

We define the ideal $I_G \subset K[x_1, \ldots, x_n, x_{n+1}]$ generated by the 2–minors of this matrix which correspond to the edges of G. More precisely,

$$I_G = (x_i x_{j+1} - x_j x_{i+1} : i < j \text{ and } \{i, j\} \in E(G)).$$

The ideal I_G is called the Hankel ideal associated with the graph G. In Chapter 2, we study algebraic and homological properties of I_G.

In Chapter 3, we extend the construction of the previous chapter to arbitrary generic Hankel matrices. Let $m, n \geq 3$ be two integers and X the generic $m \times n$–Hankel matrix. We consider G_1 and G_2 closed graphs on the vertex set $[m]$ and, respectively, $[n]$.

We define the Hankel ideal associated with the pair (G_1, G_2) as the ideal generated by the 2–minors of X of the form $g_{e,f} = [ij|k\ell]$ where $i < j, k < \ell$ and $\{i, j\} \in E(G_1), \{k, \ell\} \in E(G_2)$. We prove that $I_{(G_1,G_2)}$ coincides with the Hankel ideal I_G associated to an appropriate closed graph G on the vertex set $[m+n-2]$. Therefore, we may apply all the results of Chaper 2 in order to derive the properties of $I_{(G_1,G_2)}$ in terms of the combinatorial data of the graphs G_1, G_2.