Abstract

Let k be an algebraically closed field. Let $A = k[x_1, \dots, x_d]_{(x_1,\dots,x_d)}$ be the local ring and $\mathfrak{m} = (x_1, \dots, x_d)A$ be its unique maximal ideal. Also, assume that $\underline{a} = a_1, \dots, a_d$ is a system of parameters of A. Then we have the following inequality

$$e_0(\underline{a}; A) \geq c_1 \cdot \ldots \cdot c_d,$$

called local Bézout inequality in the affine d-space $\mathbb{A}_k^d$, where c_i is the initial degree of a_i for $i = 1, \dots, d$ in the form ring $G_A(\mathfrak{m}) \cong k[X_1, \dots, X_d]$ and $e_0(\underline{a}; A)$ is called the Hilbert–Samuel local multiplicity of $\underline{a} = a_1, \dots, a_d$ in A. Moreover, equality occurs if and only if $a_1^\star, \dots, a_d^\star$ form a homogeneous regular sequence in $G_A(\mathfrak{m})$. The main aim of this dissertation is to improve the local Bézout inequality. Note that our results are extension of the ones mentioned by Boďa–Schenzel [BS98] and Bydžovský [B48] for the case of affine plane $\mathbb{A}_k^2$.

A further direction is to understand the role of homological methods in this field. To this end, a certain variation of the Koszul complex will be investigated. That is, its cohomology, vanishing and non-vanishing will be characterized by the data from ring theory. More precisely, we presented a few criteria concerning regular sequences in the form module $G_M(\mathfrak{q})$ in terms of vanishing and non-vanishing of these cohomologies. Moreover, a deep investigation of a variation of local cohomology will be discussed in detail. We prove the artinianness of these local cohomologies, and discuss the vanishing and non-vanishing of the last among them.