Abstract

Let $G = (V(G), E(G))$ be a connected graph. The distance between two vertices $u, v \in V(G)$ is the length of shortest path between them and is denoted by $d(u, v)$. A vertex x is said to resolve a pair of vertices $u, v \in V(G)$ if $d(u, x) \neq d(v, x)$. For an ordered subset, $B = \{b_1, b_2, \ldots, b_n\}$ of vertices of G, the n-tuple

$$r(v|B) = (d(v, b_1), d(v, b_2), \ldots, d(v, b_n))$$

is called representation of vertex v with respect to B or vector of metric coordinates of v with respect to B. The set B is called a resolving set of G if $r(u|B) \neq r(v|B)$ for every pair of vertices $u, v \in V(G)$, i.e., the representation of each vertex with respect to B is unique. The resolving set with minimum cardinality is called metric basis of G. This minimum cardinality is called metric dimension and is denoted by $\beta(G)$. Notice that the i-th coordinate in $r(v|B)$ is 0 if and only if $v = b_i$. Thus in order to show that B is a resolving set of G, it suffices to verify that $r(u|B) \neq r(v|B)$ for every pair of distinct vertices $u, v \in V(G) \setminus B$.

Let G be a graph of order at least 2. Two vertices $x, y \in V(G)$ are said to doubly resolve the vertices u, v of G if $d(u, x) - d(u, y) \neq d(v, x) - d(v, y)$. A subset $D \subseteq V(G)$ is called a doubly resolving set of G if every two distinct vertices of G are doubly resolved by some two vertices in D, i.e., all coordinates of the vector $r(u|D) - r(v|D)$ can not be same for every pair of distinct vertices $u, v \in V(G)$. The minimal doubly resolving set problem is to find a doubly resolving set of G with the minimum cardinality. The cardinality of minimal doubly resolving set of G is denoted by $\psi(G)$. We have $\beta(G) \leq \psi(G)$ always. Therefore these sets can contribute in finding upper bounds on the metric dimension of graphs.

In this thesis, we have investigated the minimal doubly resolving set problem for necklace graph, circulant graph, antiprism graph and Möbius ladders. Also, in last part of thesis, the metric dimension problem has been investigated for kayak paddle graph and cycles with chord.