Abstract

The subject of this thesis is the homological study of some binomial ideals associated with graphs.

In Chapter 1, some necessary definitions and results from combinatorics and commutative algebra are presented.

In Chapter 2, we find a class of block graphs whose binomial edge ideals have minimal regularity. As a consequence, we characterize the trees whose binomial edge ideals have minimal regularity. Also, we show that the binomial edge ideal of a block graph has the same depth as its initial ideal.

In Chapter 3, we consider binomial edge ideals generated by maximal minors of the $2 \times n$-Hankel generic matrix. These ideals are associated with graphs which satisfy a specific combinatorial condition and are called closed graphs.

Let I_G be the binomial edge ideal on the generic $2 \times n$ - Hankel matrix associated with a closed graph G on the vertex set $[n]$. We characterize the graphs G for which I_G has maximal regularity and is Gorenstein. The main result of this chapter states that if G is a connected closed graph with the maximal cliques $F_1 = [a_1, b_1], \dots, F_r = [a_r, b_r]$ where $1 = a_1 < a_2 < \cdots < a_r < b_r = n$, then I_G is Gorenstein if and only if $a_2 = 2, a_{i+2} = b_i + 1$ for $1 \leq i \leq r-2$, and $b_{r-1} = n-1$.