Abstract

Let vertex and edge sets of graph G are denoted by $V(G)$ and $E(G)$, respectively. An *edge-covering* of G is a family of different subgraphs $H_1, H_2, \ldots, H_k$ such that each edge of $E(G)$ belongs to at least one of the subgraphs H_j, $1 \leq j \leq k$. Then it is said that G admits an $(H_1, H_2, \ldots, H_k)$-(edge)covering. If every H_j is isomorphic to a given graph H, then G admits an *H-covering.*

For a fixed graph H, a total labeling $\phi : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is said to be *H-magic* if all subgraphs of G isomorphic to H have the same weight.

One can ask for different properties of a total labeling ϕ. The total labeling is said to be *antimagic* if the weights of subgraphs isomorphic to H are pairwise distinct. Further restriction on the weights of subgraphs provides (a, d)-H-antimagic labelings where the weights of subgraphs form an arithmetic progression with difference d and first element a.

If graph G is a *2-connected plane* graph then the H-antimagic labeling is equivalent to d-antimagic labeling of type $(1, 1, 0)$, where weights of all faces form an arithmetic sequence having a common difference d and the weight of a face under a labeling of type $(1, 1, 0)$ is the sum of labels carried by the edges and vertices on its boundary.

In the first part of the thesis we will study the notions, notations and definitions about graphs and labeling of graphs.

In the second part of the thesis, we have three chapters on newly obtained results. In the chapters, we examine the existence of H_2^k-supermagic labelings for graphs G_2^k obtained from two isomorphic graphs G and G' by joining every couple of corresponding vertices $v \in V(G)$ and $v' \in V(G')$ by a path of length $k + 1$. We show that graphs $G^k(w)$, obtained from a graph G by joining all vertices in G to a vertex w by paths of length $k + 1$, keep super H-antimagic properties of the graph G. We also examine the existence of the $(H \square G_2)$-supermagic labelings of Cartesian product $G_1 \square G_2$, where G_1 admits an H-covering and G_2 is a graph of even order. Additionally, we show that if a graph G admits a (super) $(a, 1)$-tree-antimagic labeling then the disjoint union of multiple copies of the graph G keeps the same property.