## Abstract

A plane graph is a particular drawing of a planar graph on the Euclidean plane. Let $G(V, E, F)$ be a plane graph with vertex set $V$, edge set $E$ and face set $F$. A *proper entire t-colouring* of a plane graph is a mapping:

$$\alpha : \; V(G) \cup E(G) \cup F(G) \longrightarrow \{1, 2, \ldots, \; t\}$$

such that any two adjacent or incident elements in the set $V(G) \cup E(G) \cup F(G)$ receive distinct colours. The *entire chromatic number*, denoted by $\chi_{vef}(G)$, of a plane graph $G$ is the smallest integer $t$ such that $G$ has a proper entire $t$-colouring.

The proper entire $t$-colouring of a plane graph have been studied extensively in the literature.

There are several modification on entire $t$-colouring. We focus on a *face irregular entire k-labeling* of a 2-connected plane graph as a labeling of vertices, edges and faces of $G$ with labels from the set $\{1, 2, \ldots, k\}$ in such a way that for any two different faces their weights are distinct. The *weight* of a face under a $k$-labeling is the sum of labels carried by that face and all the edges and vertices incident with the face. The minimum $k$ for which a plane graph $G$ has a face irregular entire $k$-labeling is called the *entire face irregularity strength.*

Another variation to entire $t$-colouring is a *d-antimagic* labeling as entire labeling of a plane graph with the property that for every positive integer $s$, the weights of $s$-sided faces form an arithmetic sequence with a common difference $d$.

In the thesis, we estimate the bounds of the entire face irregularity strength for disjoint union of multiple copies of a plane graph and prove the sharpness of the lower bound in two cases. Also we study the existence of $d$-antimagic labelings for

the Klein-bottle fullerene that is for a finite trivalent graph embedded on the Klein-bottle with each face is a hexagon. In last chapter we investigate the 3-total edge product cordial labeling of hexagonal grid (honeycomb) that is the planar graph with $m$ rows and $n$ columns of hexagons.