Abstract

Semirings, one of the most natural generalization of rings and distributive lattices, were first appeared in the study of ideals of rings, by Dedekind [25] and then Vandiver[78] formally introduced this notion, in 1934. One of the oldest algebraic structure, set of all natural numbers, is also a semiring. Over the years, tremendous applications of the theory of semirings have been recorded [33], from both domains of mathematics. Several types of semirings considered by researchers with respect to their applications in different areas including optimization theory, theoretical physics and computer sciences([1], [26], [30], [32], [77]).

One of the most favorite type of semiring which was studied by algebraists during the last few years, is additively inverse semiring. The algebraic structure of inverse semiring was introduced by Karevellas[53] in 1973. In [9], Bandlet and Petrich characterized inverse semirings as a subdirect product of rings and distributive lattices. Sen[76], Ghosh[29] and Mukhopadhyay [74] and many others also considered the structure of inverse semiring.

Recently, another class of semirings which appeared in the corpus, is the class of MA-Semirings. Javed, Aslam and Hussain[49] identified this class, as a subclass of additive inverse semirings which satisfies the condition (A-2) stated by Bandlet and Petrich in [9]. They initiated the theory of commutators with its fundamental identities in MA-Semirings, which later proved to be very fruitful in investigating many concepts of ring theory. These include theory of dependent elements and free actions[50], commutativity and centralizing mappings[51] and the theory of derivations of MA-Semirings[49]. Indeed, this algebraic structure is of considerable interest in targeting and generalizing many Lie type results of rings and algebra to semirings.

As the name suggests, in this thesis, we will be considering MA-Semirings in regards of various concepts of ring theory. As usual, the first chapter will be

devoted to preliminaries that includes some basic concepts of semiring theory. The chapter contains a brief introduction to the class of MA-Semirings and the notion of commutators in MA-Semirings.

Chapter 2, deals with the theory of Lie and Jordan ideals of MA-Semirings. We introduce the notion of Jordan ideals of MA-Semiring and investigate famous results of Herstein[35, 40], in the setting of MA-Semirings. Lie ideals of MA-Semiring have been defined, already, by Javed and Aslam [51]. In this chapter, we explore Lanski[56] and Herstein's work[43] on Lie ideals and extend their work to MA-Semirings. Some results of this chapter have accepted for the publication in the Italian Journal of Pure and Applied Mathematics[71].

In Chapter 3, we study the theory of derivation of MA-Semirings. In this regard, we probe the most investigated work of Posner on derivation of prime rings [66]. We also present the proof of one of the famous Posner's theorem, namely, Posner's second theorem of derivation, for MA-Semirings. The results of this chapter have accepted for the publication in Hacettepe Journal of Mathematics and Statistics[72].

Chapter 4, will be devoted to the study of Jordan Mappings in MA-Semirings. We formulate the notion of Jordan homomorphism and Jordan triple Homomorphism of MA-Semirings. A few well-known results obtained by Brešar[15] and Herstein[38], in this subject, are also generalized for MA-Semirings. In last two sections, we define Jordan derivation and Jordan triple derivation of MA-Semirings. In this chapter, we also prove that a Jordan derivation of 2-torsion free prime MA-Semiring is a derivation, which generalizes classical result of Brešar's [13]. The contents of this chapter have published in the Journal of Open Mathematics[69].

In Chapter 5, we will study the most important concept of left centralizers on MA-Semirings. The work in this chapter, is motivated by the study of Zalar, Vukman and Brešar [19, 79, 80, 84] on left centralizers. Most of the results of this chapter are part of our publication in the Journal of Quasigroups and related systems[70] and in the Journal of Discussiones Mathematicae-General Algebra and Applications[68].

In the last chapter, we will be considering MA-Semiring with the notion of dependent elements and free actions. In his Ph.D. thesis[50], Javed introduced the notions of dependent elements and free actions for the class of MA-Semirings. This chapter is devoted for the development of these notions.