Abstract

The development of statistical process control (SPC) has evolved considerably, moving from classical Shewhart charts—effective in detecting large shifts but relatively insensitive to small ones—to memory-based schemes such as CUSUM and EWMA that enhance sensitivity to smaller changes but require parameter tuning. Later contributions introduced hybrid approaches and algorithmic methods, while recent studies have extended SPC to non-normal processes, economic designs, and Bayesian methodologies. Despite the promise of Bayesian analysis in providing flexibility and robustness through the integration of prior knowledge with observed data, challenges remain regarding prior sensitivity, loss-function specification, and computational complexity. A particularly underexplored area in this progression is the monitoring of higher-order parameters, especially the shape parameter of waiting time distributions such as the Inverse Gaussian (IGD) and Lomax distribution, which play a vital role in determining skewness, tail behavior, and central tendency. Since these distributions frequently arise in manufacturing, healthcare, and service systems, their accurate monitoring is crucial for ensuring reliability and quality improvement. However, existing literature has given limited attention to parameter uncertainty and to the use of Bayesian methods for monitoring shape parameters under realistic conditions.

This research addresses these gaps by developing both classical and Bayesian control charting schemes for monitoring the shape parameter of IGD and Lomax distributions. The study introduces novel Bayesian Shewhart, EWMA, and CUSUM charts under informative and non-informative priors, including uniform, Jeffreys, and gamma priors. By combining prior information with the likelihood functions of the IGD and Lomax distributions, posterior distributions are derived, from which Bayesian estimators, posterior risks, and control limits are constructed using a variety of symmetric and asymmetric loss functions x

such as SELF, MSELF, WSELF, DLF, KLF, and PLF. The performance of these charts is rigorously evaluated through extensive Monte Carlo simulations, considering variations in sample size, hyperparameters, and smoothing constants, and benchmarked against classical approaches. Results demonstrate that Bayesian charts—particularly those based on maximum likelihood estimation and non-informative priors—outperform classical schemes in detecting small, moderate, and large shifts in the shape parameter, thereby offering more reliable monitoring under parameter uncertainty. Moreover, the classical and Bayesian monitoring of the shape parameters of the underlying waiting time distributions is also made using simulated as well as real datasets.

Overall, this study makes a significant contribution by advancing Bayesian SPC methodologies tailored to waiting-time-driven processes, addressing critical gaps in the monitoring of shape parameters, and offering practical tools for enhancing stability, reliability, and quality in manufacturing and service industries.