Abstract

The transport of a solute under the combined effect of advection and diffusion is governed by the advection-diffusion equation. Many natural processes such as the atmospheric pollution, the sea water intrusion and thermal pollution of river systems are also modeled by the advection-diffusion equation.

Since the time-fractional derivative operator describes the memory effects, the study of mathematical models with fractional differential equations can provide interesting solutions for flows of fluids with complex rheology.

The fundamental solutions play an important role in the study of solutions of the ordinary/partial differential equations. Two new problems, based on the advection-diffusion equation with time-fractional derivative have been solved in this thesis.

Firstly, was solved the Dirichlet problem for the two-dimensional time-fractional advection-diffusion equation with Caputo-Fabrizio derivatives without singular kernel. Fundamental solutions for the fractional model are determined and, these solutions are particularized for fractional diffusion process, ordinary advection-diffusion and ordinary diffusion equation. The advection-diffusion process with a concentrated source was, also, studied. It is found that memory effects change significantly the solute concentration in the flow domain.

The second problem is referring at the analytical solutions of the fractional advection-diffusion equation with time-dependent pulses on the boundary. The fundamental solutions to the Dirichlet problem are determined using the integral transform technique. Using the Duhamel's principle, the analytical solutions to the Dirichlet problem with pulses on the boundary are determined. A particular case for the input

boundary condition is numerically analyzed.

Magneto-hydrodynamic flows of the electrically conducting fluids have applications in biological processes, in medical diagnosis and therapies and in many engineering areas. The electro-magneto-hydrodynamic flows in cylindrical domains are important for both theoretical and practical problems.

In present thesis, I studied two problems regarding some unsteady two-phase blood flows and thermal transport through cylindrical tubes.

Magneto-hydrodynamic flows of the blood, modeled as a Bingham fluid, through a stenosed artery were analyzed under the influence of oscillating pressure gradient, body acceleration and external magnetic field perpendicular to the flow direction. Two solutions are obtained, namely, an approximate series solution, determined in the assumption that the Womersley frequency parameter has a small value and, an analytical solution obtained by coupling of the Laplace transform with the finite Hankel transform. The shear stress on the stenozed wall is determined and numerically analyzed.

The theoretical study of the blood flow along with particles suspension through cylindrical capillary is made by the electro-magneto-hydrodynamic approach. The blood is modeled as a Newtonian fluid which moves under the influence of a time variable pressure gradient, an external magnetic field and an electric field. The study of thermal transport is based on the energy equation for two phase thermal transport of blood and particles suspension. The effects of viscous dissipation and volumetric heat generation due to Joule heating effect are considered. Analytical solutions for fluid velocity, particles velocity and temperature fields have been determined using the integral transform method. The influence of the electro-magnetic field on the blood flow and particles flow was analyzed in some particular cases of the pressure gradient.