Abstract

In this thesis, we explore various modular properties of elliptic genus of the product of Hilbert schemes,

$$\mathrm{Hilb}^{k_1}[\mathbb{C}^2] \times \mathrm{Hilb}^{k_2}[\mathbb{C}^2] \times \cdots \times \mathrm{Hilb}^{k_N}[\mathbb{C}^2],$$

regarding to the certain bundles E defined on the product whose weights were given in [I]. The Hilbert scheme of n points on $\mathbb{C}^2$, denoted by $\mathrm{Hilb}^n[\mathbb{C}^2]$, consist of all ideals of co-length n in $\mathbb{C}[x, y]$. We discuss the modular properties of the generating functions corresponding to the product of Hilbert schemes, obtained by summing over $(k_1, k_2, \cdots, k_N)$. We also discuss the recursive structure of these generation functions in terms of the Hecke operators. Moreover, we discuss the properties of the generating functions near some special loci in parameter space corresponding to the degeneration of the mirror Riemann surfaces.