Abstract

Various types of labelings of graphs have been intensely studied during the last few decades. One of the most challenging task in the theory of labeled graphs is to prove that it is possible to label edges of any graph other than K_2 with distinct consecutive numbers starting from 1 so that the sum of numbers at each vertex differ. Such a labeling is called antimagic.

This thesis focuses on variations of antimagic and irregular types of labelings. The common property of these types of labelings is that the associated elements weights are distinct. However, the difference is that while in antimagic types of labelings the used labels must be distinct (usually also consecutive) integers, in irregular types of labelings the labels can repeat.

Let us label the vertices of G with distinct numbers from 1 up to $|V(G)|$ and the edges with the next consecutive integers. If the labeling has the property that for every subgraph isomorphic to H in G the sums of labels of all vertices and edges belonging to H form an arithmetic progression with the initial term a and the common difference d, such a labeling is called super (a,d)-H-antimagic. In this thesis we will deal with the study of the existence of super (a,d)-H-antimagic labelings for fans when subgraphs H are cycles. We will describe super (a,d)-cycle-antimagic labelings for different cycles that cover fan graphs and for wide a range of differences.

We will introduce concepts of the face irregularity strength of type (α,β,γ) and of the same-face irregularity strength of type (α,β,γ) of a plane graph G, where $\alpha,\beta,\gamma \in \{0,1\}$. We will estimate some lower bounds and some upper bounds for these new graph invariants. We will also determined the precise values of these parameters for certain families of plane graphs.

A total labeling which assigns numbers from 1 up to k for edges and assigns even numbers from 0 up to k to vertices of a graph is called an edge irregular reflexive labeling if all edge weights are different. The weight of an edge is defined as the sum of the labels of its ends and the edge label itself. The minimal k for which such labeling exists is known as the reflexive edge strength of the graph. We will determine the precise values of the reflexive edge strength for the Cartesian product of two cycles and for the Cartesian product of two paths. We will also describe a connection between the reflexive edge strength and the total edge irregularity strength of a graph.