## Abstract

An *edge-covering* of $G$ is a family of subgraphs $H_1, H_2, \dots, H_t$ such that each edge of $E(G)$ belongs to at least one of the subgraphs $H_i$, $i = 1, 2, \dots, t$. Then it is said that $G$ admits an $(H_1, H_2, \dots, H_t)$-*(edge) covering*. If every subgraph $H_i$ is isomorphic to a given graph $H$, then the graph $G$ admits an $H$-*covering*.

Let $G = (V, E, F)$ be a plane graph admitting $H$-covering. For the subgraph $H \subseteq G$ under an entire $k$-labeling $\varphi : V(G) \cup E(G) \cup F(G) \to \{1, 2, \dots, k\}$, we define the associated $H$-weight as

$$w_\varphi(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e) + \sum_{f \in F(H)} \varphi(f).$$

An entire $k$-labeling $\varphi$ is called an $H$-*irregular entire* $k$-*labeling* of the plane graph $G$ if for every two different subgraphs $H'$ and $H''$ isomorphic to $H$ there is $w_\varphi(H') \neq w_\varphi(H'')$. The *entire $H$-irregularity strength* of a plane graph $G$, denoted by $\mathrm{Ehs}(G, H)$, is the smallest integer $k$ such that $G$ has an $H$-irregular entire $k$-labeling.

Similarly is defined a *vertex-face $H$-irregularity strength* denoted by $\mathrm{vfhs}(G, H)$ and also an *edge-face $H$-irregularity strength* denoted by $\mathrm{efhs}(G, H)$.

One of the interesting kind of labelings are cordial labelings. For a simple graph $G = (V, E)$ we deal with an edge labeling $\varphi : E(G) \to \{0, 1, \dots, k-1\}$, $2 \leq k \leq |E(G)|$, which induces a vertex labeling $\varphi^* : V(G) \to \{0, 1, \dots, k-1\}$ in such a way that for each vertex $v$, assigns the label $\varphi(e_1) \cdot \varphi(e_2) \cdot \ldots \cdot \varphi(e_n) \pmod{k}$, where $e_1, e_2, \dots, e_n$ are the edges incident to the vertex $v$. The labeling $\varphi$ is called a $k$-total edge product cordial labeling of $G$ if $|(e_\varphi(i) + v_{\varphi^*}(i)) - (e_\varphi(j) + v_{\varphi^*}(j))| \leq 1$ for every $i, j$, $0 \leq i < j \leq k-1$, where $e_\varphi(i)$ and $v_{\varphi^*}(i)$ is the number of edges and vertices with $\varphi(e) = i$ and $\varphi^*(v) = i$, respectively.

In the thesis, we estimate the bounds of the parameters $\mathrm{Ehs}(G, H)$, $\mathrm{vfhs}(G, H)$ and $\mathrm{efhs}(G, H)$, and determine the precise values of these parameters for certain families of plane graphs to demonstrate that the obtained bounds are tight.

Also we examine the existence of 3-total edge product cordial labelings for rhombic grid graphs, for toroidal fullerenes and for Klein-bottle fullerenes.