Introduction

Navigation can be studied in a graph-structured framework in which the navigation agent (which we shall assume to be a point robot) moves from node to node of a "graph space". The robot can locate itself by the presence of distinctly labeled "landmark" nodes in the graph space. For a robot navigating in Euclidean space, visual detection of a distinctive landmark provides information about the direction to the landmark, and allows the robot to determine its position by triangulation. On a graph, however, there is neither the concept of direction nor that of visibility. Instead, we shall assume that a robot navigating on a graph can sense the distances to a set of landmarks.

Evidently, if the robot knows its distances to a sufficiently large set of landmarks, its position on the graph is uniquely determined. This suggests the following problem: given a graph, what are the fewest number of landmarks needed, and where they should be located, so that the distances to the landmarks uniquely determines the robot's position on the graph? This is actually a classical problem about metric spaces. A minimum set of landmarks which uniquely determines the robot's position is called a "metric basis", and the minimum number of landmarks is called the "metric dimension" of a graph.

Motivated by the problem of uniquely determining the location of an intruder in a

network, the concept of metric dimension was introduced by Slater in [36, 37] and studied independently by Harary and Melter in [19]. Slater refereed to the metric dimension of a graph as its "location number" and motivated the study of this invariant by its application to the placement of a minimum number of sonar/loran detecting devices in a network so that the position of every vertex in the network can be uniquely described in terms of its distances to the devices in the set.

For a subset $S \subset V(G)$ and a vertex v of a connected graph G, the distance $d(v, S)$ between v and S is defined as usually, by $d(v, S) = \min\{d(v, x) : x \in S\}$. If $\Pi = (S_1, S_2, ..., S_k)$ is an ordered k- partition of $V(G)$, the representation of v with respect to Π is the k- tuple $r(v|\Pi) = (d(v, S_1), d(v, S_2), ..., d(v, S_k))$. If the k- tuples $r(v|\Pi)$ for $v \in V(G)$ are all distinct, then the partition Π is called a resolving partition and the minimum cardinality of a resolving partition of $V(G)$ is called the partition dimension of G and is denoted by $pd(G)$.

These concepts have some applications in chemistry for representing chemical compounds ([9],[25]) or to problems of pattern recognition and image processing, some of which involve the use of hierarchical data structures [31].

This thesis is divided into six chapters. The first two chapters consist of basic concepts and terminology of graphs and distances in graphs. In the third chapter, the metric dimension of plane graphs induced by some classes of convex polytopes has been determined and it was proved that these plane graphs have constant metric dimension and only three vertices appropriately chosen suffice to resolve all the vertices of these graphs.

The fourth chapter deals with the metric dimension of generalized Petersen graphs $P(n, 3)$. In this chapter, we study the metric dimension of the generalized Petersen

graphs $P(n,3)$ by giving a partial answer to an open problem raised in [8]. In the fifth chapter, d-sets for connected graphs have been defined and it is shown that for a connected graph G of order n and diameter 2 the number of pairs $\{x,y\}$ such that their d-sets are equal to $V(G)$ is bounded above by $\lfloor n^2/4 \rfloor$ and it is conjectured that this holds for any connected graph. A lower bound for the metric dimension of G, $dim(G)$ is proposed in terms of a family of d-sets of G having the property that every subfamily containing at least $r \geq 2$ members has an empty intersection. Three sufficient conditions which guarantee that a family $\mathcal{F} = (G_n)_{n\geq 1}$ of graphs with unbounded order has an unbounded metric dimension are also proposed. Finally, d-sets are used to show that $dim(Ne_n) = 3$ when n is odd and 2 otherwise, where Ne_n is the necklace graph of order $2n+2$. In the sixth chapter, we study the metric dimension and partition dimension of some infinite regular graphs generated by tilings of the plane by regular triangles and hexagons. It is shown that these graphs have no finite metric bases but their partition dimension is finite and is evaluated in some cases and it is proved that for every $n \geq 2$ there exists finite induced subgraphs of these graphs having metric dimension equal to n as well as infinite induced subgraphs with metric dimension equal to three. Also some open problems are suggested in the seventh chapter.