Mathematical properties of a spectral function of single-particle energy states in a physical system of N interacting Fermi-particles are investigated in analytical and computational way. A new approximation of an equilibrium spectral function is offered on the basis of Fourier transform of the exact Kadanoff expression, which could be generalized to the case of nonequilibrium system when disturbances slowly vary in space and time.

A new integral formula is introduced in the theory of Hilbert transform, which enters the exact expression for an equilibrium spectral function. A certain class of Hilbert pairs of functions is introduced and a theorem is offered and proved, which established sufficient conditions for the validity of the physical sum rule on the basis of the theory of Poisson integral and the Privalov-Plessner theory of boundary properties of analytical functions.