Abstract

A plane graph is a particular drawing of a planar graph on the Euclidean plane. Let \( G(V, E, F) \) be a plane graph with vertex set \( V \), edge set \( E \) and face set \( F \). A proper entire \( t \)-colouring of a plane graph is a mapping:

\[
\alpha : V(G) \cup E(G) \cup F(G) \rightarrow \{1, 2, \ldots, t\}
\]
such that any two adjacent or incident elements in the set \( V(G) \cup E(G) \cup F(G) \) receive distinct colours. The entire chromatic number, denoted by \( \chi_{vef}(G) \), of a plane graph \( G \) is the smallest integer \( t \) such that \( G \) has a proper entire \( t \)-colouring.

The proper entire \( t \)-colouring of a plane graph have been studied extensively in the literature.

There are several modification on entire \( t \)-colouring. We focus on a face irregular entire \( k \)-labeling of a 2-connected plane graph as a labeling of vertices, edges and faces of \( G \) with labels from the set \( \{1, 2, \ldots, k\} \) in such a way that for any two different faces their weights are distinct. The weight of a face under a \( k \)-labeling is the sum of labels carried by that face and all the edges and vertices incident with the face. The minimum \( k \) for which a plane graph \( G \) has a face irregular entire \( k \)-labeling is called the entire face irregularity strength.

Another variation to entire \( t \)-colouring is a \( d \)- antimagic labeling as entire labeling of a plane graph with the property that for every positive integer \( s \), the weights of \( s \)-sided faces form an arithmetic sequence with a common difference \( d \).

In the thesis, we estimate the bounds of the entire face irregularity strength for disjoint union of multiple copies of a plane graph and prove the sharpness of the lower bound in two cases. Also we study the existence of \( d \)- antimagic labelings for
the Klein-bottle fullerene that is for a finite trivalent graph embedded on the Klein-
bottle with each face is a hexagon. In last chapter we investigate the 3-total edge
product cordial labeling of hexagonal grid (honeycomb) that is the planar graph with
$m$ rows and $n$ columns of hexagons.