ABSTRACT

The structure theory of *l*-ring was introduced by remarkable work of Birkoff and Pierce in 1956[6]. They proved that the *l*- radical $N_l(R)$ of *l*-ring R is the union of all *l*-nilpotent ideals. However, the structure of *l*-rings was enhanced by D-G Johnson in 1959[23] Much of his research related to structure theory upon the certain class of *l*-rings (*f*-rings). In this connection, he developed the theory of Jacobson radicals and established the different characterizations of Jacobson radicals. He was able to develop different relation between $N_l(R)$ and $J_l(R)$ in different environment. Laterly, Johnson's work was extended by various authors, Shatalova[34] ,Steinberg[35-43], Anderson[2,3], Brainerd[7], Hayes[14] on *l*-rings. D.E. Diem[9] introduced the notion *l*-prime radicals, $P_l(R)$, as the intersection of all *l*-prime ideals in R. He was able to develop different relation between the $P_l(R)$ and. $N_l(R)$ under the certain conditions. H-J-Shy and Viswanathan [33] introduced certain classes of *l*-rings (i.e. Archimedean, *ps*-rings) and proved that $N_l(R)$ is equal to $P_l(R)$ in these classes. Boris [27] established for a given well ordered Archimedean ring R, there is unique *l*-nilpotent ideal which is equal to $N_l(R)$. Moreover, the *l*-neotherian , *l*-artinian rings was extensively studied by Anderson [3], Jingjing [21]. Stuart A. Stinberg[43] developed matrix equation i.e. $(J_l(R_n)) = (J_l(R))_n$ and introduced the *l*-quasi regularity in *l*-rings and established its fundamental theorems.

The purpose of this dissertation is to study the radical theory of *l*-rings.

In chapter I, we include the necessary preliminaries of theory of *l*-rings, required for the development of the next chapters.

In chapter II, we discuss the notion of *l*-nill radicals, $N_l(R)$ due to Birkhoff[6], and reproduced some results of Birkohff which are available in his paper [6] in quite implicit form and has shown that $N_l(R)$ is the union of all *l*-nilpotent ideal of R.

Diem [9] introduced the l-prime radicals and establish its fundamental theory. However much of his work was on non-commutative class of l-rings. However, in the class of l-commutative rings we are able to show that $N_l(R) = P_l(R)$, which is helpful to establish a few results on l-semi ideals. Diem[9] introduced the notions of l-domains and positive zero divisors in canonical way. By using the theory of positive zero divisors, we are able to generalize a few results of Johnson [23]. All these are established in chapter III.

In chapter IV, we discuss the notion of l-Jacobson radicals and reproduce some results which are available in literature in quite implicit form. The notion of l-primitivity was introduced by Johnson[23]. However, the theory of l-primitivity was not much explored. We are able to develop some relationship between the l-annihilators in term of l-modules. In this way we are able to define l-primitive ideals in term of l-annihilators of certain l-simple modules. We are also show that $J_l(R)$ is intersection of all l-primitive ideals. Consequently, we establish l-Jacobson radical is the collection of all l-annihilators of l-simple module over R.

In chapter V, our effort is to introduce general radical theory of l-rings.