ABSTRACT

Radical theory is one of the most important tools in studying the structure of rings. The first known radical is the "nilpotent radical" which appeared in the study of hyper complex numbers and later its abstract form was defined by Wedderburn in [64]. Koethe generalized the idea of nilpotency by introducing nility. He was able to show that if N(R) is a maximal nil ideal of a ring R, then R / N(R) has no non-zero nil ideal. The notion of nilpotency fails to show the Koethe's last property for nility.

In [9] and [29] new radicals were introduced by Brown-Mecoy and Jacobson respectively, which also satisfied Koethe's properties for nility. These were the first signs of abstract theory. Finally general theory of radicals for rings was introduced independently by Amitsur ([2], see also[3,4,5]) and by Kurosh ([33]).

The Jacobson radicals were extended by S. Bourne [11] for semiring. The notion of Brown Mecoy radicals for hemirings was introduced by D. Lattore [39]. It was shown that nil radical and Brown Mecoy radicals coincide over commutative hemirings.

D. M. Olson and T. L. Jenksins [49] introduce general radical theory for hemirings and obtained a few results corresponding to the general radical theory of rings. In this dissertation, we further explore and generalize many useful results of radical theory of rings. By introducing some specific notations and symbols,we make the subject lucid and simple.

Chapter 1 includes some necessary preliminaries to make the dissertation self contained.

In chapter 2, we reproduced a few results of [49] and establish them slightly different and easy way by using some specific notation and symbols. We also generalize a few results of D. M. Olson and T. L. Jenksins [49]. Moreover a few new results associated to the semisimple classes are established.

Chapter 3 is devoted to the construction of new radical classes for hemirings. In this connection, we give the construction of lower radical

classes and upper radical classes of hemirings. Different constructions for lower radical classes for hemiring are introduced and established in natural way. We establish the necessary and sufficient conditions for hereditary of upper radical classes of hemirings. The lower radicals are further studied by introducing the Q condition for hemirings and generalize a few results of [49].

In chapter 4, we discuss the extended sum of radical classes of hemirings. The notion of sum of two radical classes of ring has been introduced by Y. L. Lee and R. E. Propes [42] and this concept was further extended and generalized by R. E. Propes and A. M. Zaidi [53] in the form of extended sum. Here, we extend it to classes of hemirings. We not only develop the ring theoretical analogy, but also generalize a few results of [53] by droping the extra conditions (see theorem 2) for hemirings and hence for rings.

In chapter 5, we study the upper radical classes for hemiring by introducing the notion of middle class for hemiring. A few open problems are also discussed in 5.2.