ABSTRACT

Let X be a non empty set and consider the set theoretic – difference in the power set P(X). One can observe that the following properties hold in P(X) (i) $(A - B) - (A - C) \subseteq C - B$ (II) $A - (A - C) \subseteq C$ (III) $A \subseteq A$ (IV) $\varphi \subseteq A$ (V) $A \subseteq B$, $B \subseteq A$ imply that $A = B$, where $A \subseteq B$ iff $A - B = \varphi$ for all A , B , C in P(X). These properties of the set theoretic difference were axiomatized by Iseki and Imai in 1966, and were used as a starting point for new type of algebras known as BCK-algebras. Another motivation for the definition of BCK-algebras comes from Meredith's BCK-implicational calculus (see A.N Prior [23]). Theory of BCK-algebra began with the fundamental work of Iseki and Imai [19] as algebraic generalization of BCK-propositional logic in 1966 as much as the same way as Boolean logic was formalized as Boolean algebra. Over the past three decades the subject has been developed from algebraic point of view (see [19, 20, 26, 27]) and lot of work has been produced dealing with the general theory, categorical aspect and ideal theory of BCK-algebra (see [4, 8, 17, 20]). Several efforts have been accomplished to develop the relationship of BCK-algebras to other structures as such as D-Posets [21], MV-algebra [22], QVM-algebras [13]. BCK–module was defined by M. Aslam, H. A. S. Abujabal and A. B. Thaheem in 1994 [1] as an action of a BCK–algebra over a commutative group. It

was shown in [1], every implicative BCK–algebra forms a BCK – module over itself.

The main aim of this thesis is to initiate and investigate the further Homological aspects of BCK–module and general module theory. The thesis is organized as follows.

In first chapter we recall some fundamental concepts and results from the theory of BCK-algebras that we require for the development of module theory of these algebras in the following chapters. In addition, we include some basic facts from the theory of ordered structures and establish our notations and terminology used in the sequel.

In second chapter, the notion of BCK-module has been discussed. In fact, the concept of BCK-modules was introduced in [1]. In this chapter we review the above paper and reproduce the results of the paper for the reader convenience, and establish isomorphism theorems. Moreover, we prove new results (Lemma 2.3.3, Lemma 2.3.4, Theorem 2.3.6 and Theorem 2.3.7) of composition of homomorphism used in the sequel.

In third chapter, we introduce the notion of chains of BCK- modules and define necessary and sufficient conditions for BCK-modules to satisfy the maximal and minimal conditions. Moreover, we establish Zassenhaus Lemma, Schrier refinement theorem and Jordan Holder theorem for

In fourth chapter, we study homological aspects of BCK – modules. In this connection, Injective and Projective modules are discussed. Moreover, a few new results related to exact sequence and commutative diagrams such as (Theorems 4.2.4, 4.2.5, 4.2.7, 4.2.8, 4.2.9, 4.2.10, 4.2.11, 4.3.1, 4.3.2, 4.4.1, 4.4.2) are proved.