ABSTRACT

Let X be a normed space, $Y \subseteq X$, we say that f has fixed point property if every continuous function f from Y to Y has fixed point. If X is a finite dimensional nromed space then $B(X)$ has fixed point property. However, in infinite dimensional $B(X)$ lose this property due to lack of compactness. This leads to investigate appropriate substructure of infinite dimensional normed space having fixed point property. In this connection, a lot of research is devoted (see [7,8,13,14,15]). For example, every convex compact subset of normed space has fixed point property (see [2.2.31,2.2.35]). The second approach to investigate fixed point property is to upgrade continuity referred as contraction conditions.

In chapter 1, some necessary preliminaries related to normed space are given which are required in sequel. In this connection, some important fixed point theorems are also presented due to their extensive applications in different discipline of Mathematics.

It is well known that if X is finite dimensional normed space then $\overline{B(X)}$ has fixed point property (see [1.7.2]). A remarkable result to determine dimension of normed space say that, X is finite dimensional iff $\overline{B(X)}$ is compact. This means that in infinite dimensional normed space $\overline{B(X)}$ does not remain compact. In order to investigate fixed point in certain subset of infinite dimensional normed space some appropriate condition imposed including convexity, compactness, boundedness (see [1.7.1,2.3.1,2.2.30]). Therefore, it is reasonable to study some topological properties of normed space. The second chapter is devoted to study these aspects of normed space. The notions of retraction, homotopy, retract and contractibility are modern tools to study fixed point theorem in normed space.

In chapter 3, we study fixed point theorems related to Reich function, compatible mappings and reproduced their proof. Moreover, we also studied the fixed point theorems related to demi-closed mappings. In this connection, we are able to extend a few results of K.T.Ravindran and Anoop S.K.(see [14]) in a canonical way.

In chapter four, we study the fixed point theorems in vector space, with appropriate generalized distance, which admits a topology and is continuous in its nature. This is indeed useful to study the fixed point theorems in topological vector space. In this connection, we prove some fixed point theorems in 2-Banach spaces.