ABSTRACT

In this thesis two methods are developed, based on rational approximations to the matrix exponential function, for solving heat equation with variable coefficients. These methods are L-stable, fourth-order accurate in space and time, and do not require complex arithmetic. In the development of these methods second order partial derivatives are approximated by fourth-order difference approximations which give a system of ordinary differential equations and that system is expressed in matrix-vector forms. Also the solutions of these systems satisfy recurrence relations that ultimately lead to the development of parallel algorithms. These algorithms are tested on heat equations with variable coefficient, subject to homogeneous and time-dependent boundary conditions, and no oscillations are observed in the results.

CHAPTER ONE contains some general topics which are helpful in rest of the thesis, for example differential equations and their classes, Taylor's formula and analysis of finite difference methods: stability, convergence etc.

CHAPTER TWO covers historic background of certain numerical methods for solving heat equation. In this chapter we have explained Method of Lines, some rational approximations for matrix exponential function, further the techniques of avoiding complex arithmetic, which may exist in numerical methods.

In CHAPTER THREE, numerical methods with homogeneous boundary conditions and time-dependent boundary conditions of one-dimensional heat equation with variable coefficients are introduced. Further we have checked the L-stability of these methods. Parallel algorithms for heat equation with variable coefficients are developed and presented in tabular forms.

In CHAPTER FOUR, the physical interpretations of the methods developed in chapter three are exercised. Maximum errors for different numerical problems are given in tabular forms. Analytical and numerical solutions of the given examples are also presented in this chapter.