ABSTRACT

Many authors studied the commutativity of prime and semiprime rings equipped with non-zero derivations. Posner [96] proved that if *R* is a prime ring, then the existence of a nonzero derivation $d: R \to R$ such that $[d(x), x] = 0$ forces ring to be commutative.Herstein [66] proved that if a non-zero derivation *d* on a prime ring *R* with $charR \neq 2$ satisfies $[d(x), d(y)] = 0 \ \forall x, y \in R$,then *R* is a commutative.Daif and Bell [49] proved that a semiprime ring *R* will be commutative if it admits a derivation *d* such that $xy + d(xy) = yx + d(yx) \ \forall x, y \in R$ or $xy - d(xy) = yx - d(yx) \ \forall x, y \in R$.Daif[50] also studied commutativity for semiprime ring *R* with derivation *d* satisfying $d(x)d(y) + d(xy) = d(y)d(x) + d(yx) \ \forall x, y \in R$.In this thesis we will study derivation *d* which satisfies the condition $d(xy) = d(yx) \ \forall x, y \in R$ and prove that the existence of such non-zero derivations on a prime ring *R* forces ring to be commutative. We establish some fundamental and useful identities of O-symmetric derivations. With the help of these identities, we are able to establish a relationship between O-symmetric derivations and commuting derivations on semiprime ring. We also prove that if O-symmetric derivation on a prime ring with unity is zero at a non-trivial idempotent of the ring, then it vanishes on the whole ring. The commuting conditions for an O-symmetric derivation on a given ring are also be investigated.

We also introduce derivation $d: R \to R$ such that $d(x^2) = (d(x))^2 \ \forall x \in R$ and establish some identities . We will also introduce the notion of Jordan prime rings and Jordan zero rings and establish a relation between zero ring and Jordan zero ring.