ABSTRACT

The importance of approximation techniques for the numerical solution of heat equation with constant coefficients can not be denied. In this thesis two numerical methods, based upon rational approximation to the matrix exponential function, for solving heat equation are developed. These methods are L-stable, fifth-order accurate in space and fourth-order accurate time i.e. $O(h^5, l^4)$ and do not require complex arithmetic. In the development of these methods second-order partial derivatives are approximated by fifth-order difference approximations which gives a system of ordinary differential equations and that system is expressed in matrix-vector forms. Also the solutions of these systems satisfy recurrence relations that ultimately lead to the development of parallel algorithms. These algorithms are tested on heat equation subject to homogeneous and time-dependent boundary conditions, and no oscillations are observed in the results. Also for the mathematical calculation MATLAB is used.

CHAPTER ONE contains some general topics which are needed in later chapters, for instance, derivation of heat equation, Taylor formula and Taylor series, analysis of finite difference methods, local truncation error, stability and convergence.

CHAPTER TWO covers historical background of certain numerical methods for solving heat equation. In this chapter we have explained Method of Lines, some rational approximations for matrix exponential function, further the techniques of avoiding complex arithmetic, which may exists in numerical methods.

In CHAPTER THREE, numerical methods with homogeneous and non-homogeneous boundary conditions of heat equation with constant coefficients are introduced. The matrix exponential function is approximated by the rational approximation. L-stability is introduced in section 3.3 and parallel algorithms for heat equation are developed and presented in tabular forms.

In CHAPTER FOUR, we use the methods which are developed in CHAPTER THREE and solve six problems. The Maximum errors for these six problems are given in tabular form. Also we compare the errors of these problem b/w fourth-order method and those methods which are developed in chapter 3 and conclude, these Methods are most accurate than fourth-order method.