ABSTRACT

Coset diagrams for the orbit of the modular group $G = \langle x, y: x^2 = y^3 = 1 \rangle$ acting on real quadratic fields give some interesting information. In [13] by using these coset diagrams it has been shown that for a fixed value of n, a non-square positive integer, there are only a finite number of real quadratic irrational numbers of the form $\alpha = (a + \sqrt{n})/c$ where α and its algebraic conjugate $\bar{\alpha} = (a - \sqrt{n})/c$ have different signs called ambiguous numbers and that part of the coset diagram containing such numbers forms a single circuit (closed path) and it is the only circuit in the orbit of α. If $n = k^2m$, where k is a non – square integer and m is square free positive integer, then a subset

$Q^*(\sqrt{n}) = \{(a+\sqrt{n})/c: a, b=(a^2 - n)/c \text{ and } c \text{ are integers and } (a, b, c) = 1\}$ of $Q(\sqrt{m})$ is a G – subset of $Q(\sqrt{m})$.

M. Aslam , S. M. Husnine and A. Majeed [11] have determined the cardinality of the set of all ambiguous numbers of $Q^*(\sqrt{n})$ as a function of n, for each non – square n. In [12], it has been shown that for each non – square positive integer n > 2, the action of G on $Q^*(\sqrt{n})$ is intransitive.

In this thesis we have discussed a classification of the elements of $Q^*(\sqrt{n})$ with respect to congruence modulo 16 and then by using these classes we have determined the G–subsets of $Q^*(\sqrt{n})$ for each non – square n.