ABSTRACT

The celebrated result known as Wedderburn-Artin theorem [47] says that every simple ring (with unity) having minimal non-zero left-ideal is of the form $M_n(D)$. This result was further generalized by Jacobson in the form of Jacobson density theorem for the class of primitive rings [47]. By using the sub-direct product one can further generalize it for the class of semi primitive rings. This process of generalization from semi primitive rings to arbitrary rings leads one to the study of various radicals which are indeed certain ideals which makes the ring much easier to analyze. Thus the radical is "the abstraction" in the structure theory of rings. For example, the Jacobson radical J(R) (intersection of all maximal ideals) of a ring is 0 if and only if R is semi primitive ring. If $J(R) \neq 0$, then one can examine the radicals in order to analyze the structure of the ring. In this connection, one may focus his attention on prime radical of the ring R (intersection of all prime ideals) which is "0" if and only if R is semi prime ring(see [6]). In this prospective, one can conclude that the work of Wedderburn (1908) [47] is the source of motivation for radical theory. In 1930 Kothe introduced the nil radical in his fundamental paper in the next two decades prominent algebraist introduced several concrete radicals, Bears lower radical, Jacobson radical etc. Amitsur and Kurosh introduced the notion of general radicals independently and establish fundamental results of first order theory. Further mile stones of radical theory were the papers of Andrunakievich in 1958 and of Anderson, Divinsky and Sulinski [33]. One of the celebrated theorem in radical theory is associated to there names as Anderson-Divinsky-Sulinski theorem which play a key role in the development of general radical theory. The first book on radical theory of rings was written by Dvinsky [6] in 1965. Later on, radical properties were associated with relevant classes of rings and refer in literature as radical classes. This approach was initiated by Leavitts in his lecture notes, which was compiled in1970. This approach was continued by Wiegandt [46] published in 1975 and Szasz [30].

This thesis provides an exposition of the structure of Radical theory of rings and its utility in Polynomial rings. Much emphasize is given on Amitsur property, Polynomial

extensibility as well as polynomial equations of radicals of rings. We extend this idea to the sum of radical classes of rings.

This thesis is organized as follows.

Chapter 1 deals with the preliminaries, related to the theory of rings that will be needed in the development of radical theory of rings in the subsequent chapters. Further, some basic concepts of radical theory of rings are also included here.

Chapter 2 is devoted to the study of radicals of polynomial rings and investigated the situation of coincidence of these radicals with others. Some results are reproduced which are available in literature in implicit form.

In **Chapter 3**, we study Amitsur property, λ-Amitsur property of radicals of polynomial rings. We introduce the notion of generalized metric extensibility in the theory of radicals. In this connection, we are able to establish some new results related to generalized metric extensibility and polynomial equation of radicals.

In **Chapter 4**, we discuss the concept of sum of two radical classes of rings, which was introduced by Yu-Lee [22]. We discuss the matrix equation for the sum of two radicals and reproduced some results established by M. Aslam and A. M. Zaidi [3]. Moreover, some other results for the sum of two radical classes are provided for the sake of completeness.

In **Chapter 5**, we establish a few results related to polynomial rings for the sum of radical classes. In this regard, we introduce the notion of semi-simple class, Amitsur property, polynomial extensibility and polynomial equations for the sum of radical classes. In the end, an open problem related to over work has been proposed.