

# Abstract

The main aim of this thesis is to introduced the study of some well-known graphs by using arithmetic function. Some classes of graphs and their properties are discussed. Let  $m$  be any positive integer and  $d_{\Lambda_m}(m) = \{d_i : d_i|m\}$  is the set of all possible divisors of  $m$  and  $|d_{\Lambda_m}(m)| = \tau_{\Lambda_m}(m)$ ,  $\tau_{\Lambda_m}(m)$  is number of possible divisors of  $m$ . By using these divisors, a new graph is generated which is called Anti Euler Totient Function Graph(AETFG) for Divisors and denoted by  $\Lambda_{\gcd}(m)$ . Furthermore the graph  $\Lambda_{\gcd}(m) = (V_m, E_m)$ , where  $V_m$  is vertex set consist of all positive divisors of  $m$  greater than 1,  $E_m$  is edge sets of two diffrent forms that is: (i) let  $u$  and  $v$  are two vertices there is edge between if  $\gcd(u, v) \neq 1$ . (ii) let  $u$  and  $v$  are two verties there is edge between if  $\gcd(u, v) = 1$ .

The main results includes the computation of no. of edges, Distance, Eccentricity, Diameter, Vertex and Edge connectivity of several classes of graphs and their Minimal vertex covering.

Another new type of graph is also introduced. Let  $m_1, m_2, m_3, \dots, m_r \in \mathbb{N}$  and  $X_i = \{1, 2, 3, \dots, m_i - 1\}$ ,  $A_i \subseteq X_i$  where  $A_i = \varphi(m_i)$  the graph generated from characteristics function is denoted as  $\Lambda_\chi(m_i)$  and futher  $\Lambda_\chi(m_i) = (V_m, E_m) = \Lambda(m_i)$  name of this graph is Characteristics Function Graph(CFG)  $V_m$  is vertex set consist of  $\chi(x_1, x_2, \dots, x_r) = (1, 0, \dots, 0)$ ,  $x_1 \in A_1$ ,  $x_i \notin A_i$ ,  $2 \leq i \leq r$ ,  $E_m$  is contain edge between two vertices if in order pair change in only single place like  $u = (1, 0, 0, \dots, 0)$ ,  $v = (1, 1, 0, \dots, 0)$  see that there is change only single place, in this frame discuss isolated, path, cycle, cubic etc and their combinatorial properties.