Abstract

A plane graph is a particular drawing of a planar graph on the Euclidean plane. Let $G(V, E, F)$ be a plane graph with vertex set V, edge set E and face set F.

The *proper entire t-colouring* of a plane graph is a mapping

$$\alpha : \; V(G) \cup E(G) \cup F(G) \longrightarrow \{1, 2, ..., \; t\}$$

such that any two adjacent or incident elements in the set $V(G) \cup E(G) \cup F(G)$ receive distinct colours. The *entire chromatic number*, denoted by $\chi_{vef}(G)$, of a plane graph G is the smallest integer t such that G has an entire t-colouring.

The proper entire t-colouring of a plane graph have been studied extensively in the literature. There are many deep results and also many challenging open problems.

If we consider a mapping

$$\phi : \; V(G) \cup E(G) \cup F(G) \longrightarrow \{1, 2, 3, \; ..., \; |V(G)| + |E(G)| + |F(G)| \; \}$$

in such a way that each vertex, edge and face receives exactly one label and each member is used exactly once as a label then the mapping ϕ we call the *labeling of type* $(1,1,1)$, LT$(1,1,1)$ for short.

The *weight* of a face under a labeling ϕ is the sum of labels carried by that face and the edges and vertices on its boundary. A labeling ϕ of a plane graph G is called *d-antimagic* if for every number s, the s-sided face weights form an arithmetic sequence having a common difference d.

It is interesting to study this concept of d-antimagic labeling of plane graph also for graphs embedded into the torus and the Klien-bottle.

In the first part of the thesis we will introduce the definitions of basic notions on colouring and antimagic labelings and we will present several known results on these types of valuations.

In the second part of the thesis we will extend the results on d-antimagic labelings for Klien-bottle polyhexes.