Abstract

In Chapter 1 we are concerned with the binomial edge ideal of a complete graph and the line graph. In the first section we get the properties of the binomial edge ideal of a complete graph: $I_2(X)$ is a prime ideal, in other words it is a toric ideal, $I_2(X)$ is Cohen-Macaulay of $\dim S/I_2(X) = n+1$, and the generators of $I_2(X)$ form a Gröbner basis for any monomial order. We also showed that $\text{in}_< I_2(X)$ and, consequently, there $I_2(X)$ have a linear resolution there $<$ denotes the lexicograpic order on S induced by the natural ordering of the vertices. For the line graph G, we showed that generators of J_G form a Gröbner basis with respect to the lexicograpic order. We determine the Betti numbers of S/J_G, $\text{depth}(S/J_G)$, the regularity, the Hilbert series etc. In the third part we present a combinatorial characterization of closed graphs given in [3] which will be useful to characterize the closed graph whose binomial edge ideal is Cohen-Macaulay.

In Chapter 2 we review the primary decomposition of binomial edge ideals from [7]. Next, we give some examples. It follows that the cycle C_n is not unmixed for $n \geq 4$. We also determine the minimal primes of line graph.

In Chapter 3 we review on of the main results of [3], namely, we present the characterization of Cohen-Macaulay binomial edge ideals of closed graphs and we present the proof of the following statement: if J_G is Cohen-Macaulay, then $\beta_{ij}(J_G)$ $= \beta_{ij}(\text{in}(J_G))$ for all i, j.

The last two chapters of my thesis consists of two annexes, namely Annex A and Annex B. In Annex A we displayed, for all graphs with 5 vertices, the following invariants : $\dim(S/J_G)$, $\text{depth}(S/J_G)$, the minimal primes and we discussed Cohen-Macaulayness and unmixedness. All the calculations were done with **Singular**. In Annex B we presented the basic facts about minimal graded free resolution, numerical data arising from minimal graded free resolutions, the Koszul complex, the Koszul complex and depth, and ideals with linear quotients.